Problem Set II

MS 105/ MS 103: Mathematics II

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1. For $h = 1, \frac{1}{2}$ and 2, Evaluate :

   a) $\triangle^2 \cos 2x$ b) $\nabla^n(e^{2x+3})$ c) $\triangle^2 \left(\frac{5x+12}{x^2+5x+16}\right)$

   d) $\nabla^2\left(\frac{1}{x^2+5x+6}\right)$ e) $\triangle^n\ (x^n)$ f) $\triangle \tan^{-1}\left(\frac{x-1}{x}\right)$.

   g) $(\triangle + \nabla)^2(x^2 + x)$

2. Using both Lagrange's interpolation formula and Newton's divided difference formula, find a polynomial that fits the following data. Hence find $f(-3)$, $f(16)$ and $f(11)$.

   a)

x :	-1	0	2	3
$f(x)$:	-8	3	1	2

   b)

x :	-2	1	0	2
$f(x)$:	3	-3	1	-1

   c)

x :	1.2	2.1	2.8	4.1	4.9	6.2
$f(x)$:	4.2	6.8	9.8	13.4	15.5	19.6

   d)

x :	-3	-1	0	2	3
$f(x)$:	-9	5	3	11	33

   e)

x :	0	1	2	4	5	6
$f(x)$:	1	14	15	5	6	19

   f)

x :	-3	-1	0	3	5
$f(x)$:	-30	-22	-12	330	3458

3. Construct the forward and backward difference table for the given data. Also find the corresponding forward and backward interpolating polynomials.

   a)

x :	-4	-2	0	2	4	6
$f(x)$:	-67	-9	1	11	69	223

   b)

x :	-3	-2	-1	0	1	2
$f(x)$:	-2	-4	-4	-2	2	8

   c)

x :	-3	-2	-1	0	1	2	3
$f(x)$:	13	7	3	1	1	3	7

   d)

x :	-4	-2	0	2	4	6
$f(x)$:	261	19	1	15	253	1291

4. The equation $f(x) = \ln x - x + 3 = 0$ has a root in the interval $(4, 5)$. Obtain the root correct to three decimal places using

i) Secant method with $x_0 = 4$ and $x_1 = 5$,
ii) Regula-falsi method with $x_0 = 4$ and $x_1 = 5$,
iii) Newton-Raphson method with $x_0 = 4.5$.

5. Using three iterations of the Newton-Raphson method, obtain approximate value of

a) $\sqrt{31}$ b) $(13)^{\frac{1}{3}}$ c) $(101)^{\frac{1}{3}}$ d) $\frac{1}{17}$.

6. Use the Bisection method to find solutions accurate to within 10^{-4} for the following problems:

a) $3x - e^x = 0$, for $1 \leq x \leq 2$.
b) $x^3 - 7x^2 + 14x - 6 = 0$, for $0 \leq x \leq 1$, $1 \leq x \leq 3.2$ and $3.2 \leq x \leq 4$.
c) $e^x - x^2 + 3x - 2 = 0$, for $0 \leq x \leq 1$.
d) $2x + 3\cos x - e^x = 0$, for $0 \leq x \leq 1$.
e) $2x\cos(2x) = (x + 1)^2$, for $-3 \leq x \leq -2$ and $-1 \leq x \leq 0$.

7. Using Newton-Raphson method and secant method to find the solutions accurate to within 10^{-5} for the following problems.

a) $e^x + 2^{-x} + 2\cos x - 6 = 0$, for $1 \leq x \leq 2$.
b) $\ln(x - 1) + \cos(x - 1) = 0$, for $1.3 \leq x \leq 2$.
c) $2x\cos 2x = (x - 2)^2 = 0$, for $2 \leq x \leq 3$ and $3 \leq x \leq 4$.
d) $(x - 2)^2 - \ln x = 0$, for $1 \leq x \leq 2$ and $e \leq x \leq 4$
e) $e^x - 3x^2 = 0$, for $0 \leq x \leq 1$ and $3 \leq x \leq 5$
f) $\sin x - e^x = 0$, $0 \leq x \leq 1$, $4 \leq x \leq 4$ and $6 \leq x \leq 7$.